# NOTES AND MEMORANDA

## ONE OF THE PHYSICAL FOUNDATIONS OF ECONOMICS

So many attempts have been made to resolve economics into a psychological science as to make it seem advisable to call attention, in some detail, to some of the physical factors which are likely to remain unaffected by psychological changes, even assuming that these psychological changes are, first, possible, and second, desirable. Some of these physical factors are so fundamental as to give character to the science of economics under all possible conditions. It seems probable that, so far as these physical factors can be ascertained, they would have to be reckoned with under any form of social organization, and with any institutional background. In certain respects, therefore, communism and capitalism, socialism and individualism, would look very much alike if they ever got into working condition, because they would have to meet certain permanent and unchangeable conditions.

It is obvious that the prime factors of production are but two in number, nature and man. There is no reason why we cannot extend this assertion to the other departments of economics, and in fact to all the other fields whose objects are the goal of human research. Production requires relations between the various parts of nature, between man and nature and between the various types of men. Whatever be the relations, there are only these two terms implied, man and nature. When economics uses these terms, it must assume their essential characters, their reactions, and their interactions, as discovered by the different departments of positive science. Economics assumes these discoveries, for it is not the

business of the economist to engage in physical and biological research, except in so far as the scientist has not concerned himself with the phenomena which are of interest to the economist. All this may appear to be a truism, and yet it has not infrequently been forgotten. As Böhm-Bawerk says at the beginning of his *Positive Theory*, even those economists who preface their works with statements of physical fact, soon forget their declarations; these statements are mere ornaments, having no connection with the rest of the work. Now and then there appears one who insists upon a firm foundation before further investigation.

There are certain facts concerning production which, as Mill remarks, are true whether we like them or not, because they are inherent in the nature of physical things. (1) The law of diminishing returns is based, as I shall try to show, upon chemistry and physics, and, like certain chemical and physical laws, is capable of being reduced to a phase of the law of probabilities. (2) The primary advantages of a division of labor are not due to any preference of ours, or to the institutions that we have seen fit to adopt. (3) The Ricardian law of rent is based upon physical variations in land as well as upon diminishing returns from land of a given grade. (4) The laws of population, while in part psychological, have still a substantial physical and physiological basis and (5) physiology, as well as psychology plays a large part in determining the laws of consumption. In a general and fundamental sense, there are certain ways of doing things that are more economical than others regardless of social or legal institutions, and of popular philosophies, fashions, etc.

The law of diminishing returns probably has more ramifications in the field of economics than any other single generalization, therefore I shall elaborate this law as a sample of the physical laws which underlie the science of economics. This law has been the object of attack especially of those whose interest in economics is secondary to their interest in social reform. It is therefore most important for us to distinguish carefully between the law itself and any conclusions that may have been drawn from it thus far, whether right or wrong.

From the older treatises on economics, the impression is given that diminishing returns is applicable only to the special case of agriculture. It is due particularly to Professor Carver that this law has become more general, and made applicable to any group of factors in production. He has insisted that this is no fleeting social phenomenon, but is a fact which has its roots in physics and chemistry. It is perhaps impossible to trace all the manifestations of this law to any one all-inclusive cause, but there is reasonable ground to suppose that it is traceable in great part to one fact which permeates all branches of science and even all activities of life. In the following attempt at a scientific explanation of this law, the procedure will be from the abstract to the concrete.

Let us take any fixed number of entities of a particular type which we may call $x$; then let us add to this successively increasing quantities of another type of entity which we may call $y$. Every time an addition is made let all the entities be paired off indiscriminately. Concerning the character of the resulting pairs as the number of $y$ entities is increased, we can lay down this general proposition, that the number of mixed pairs, those composed of an $x$ and a $y$, will increase as successive increments of $y$ are added, but this increase in the number of mixed pairs will not be proportional to the increase in the number of $y$ entities; the ratio of increase will be a diminishing one.

Let us next consider this case quantitatively. Suppose the number of $x$ entities be 5, and the number of $y$ entities added to this vary from 1 to infinity. If $1y$ is added to the $5x$, there will be 3 pairs of which 1 pair is certain to be mixed. In this case an addition of $1y$ gave us a product of one mixed pair, which is both the most and the least that can be expected. In mathematical language, the probability is here certainty, in practical language we have 100 per cent efficiency. If now we add $2y$ to $5x$, there will be a total of 3 pairs, of which we may have a maximum of 2 and a minimum of 0 mixed pair, but the probable number, or the average in the long run, will be something less than two mixed pairs but more than one. Thus by adding $1y$ we have not increased the number of mixed

pairs by 1, but by something less than 1; whereas in the first case an addition of $1y$ gave a certain result of 1 mixed pair. Continuing in this way we find that if we increase the number of $y$ up to infinity, the greatest probable number of mixed pairs will be something less than 5. The following table may make this clearer:

| No. of $x$ | No. of $y$[1] | Maximum possible number of mixed pairs | Probable number of mixed pairs | Probability in per cent of each $y$ meeting an $x$ to form a mixed pair |
|---|---|---|---|---|
| 5 | 1 | 1 | 1 | 100 |
| 5 | 3 | 3 | 2.14 | 71.43 |
| 5 | 5 | 5 | 2.78 | 56 |
| 5 | 7 | 5 | 3.18 | 46 |
| 5 | 9 | 5 | 3.46 | 38 |
| 5 | 11 | 5 | 3.67 | 33 |
| 5 | 21 | 5 | 4.20 | 20 |
| 5 | 95 | 5 | 4.80 | .5 |
| 5 | 995 | 5 | 4.97 | .5 |
| 5 | 999995 | 5 | 4.999998 | .000005 |

It will now be apparent to those who know mathematics that the above is only an application of the theory of probability. Here pure probability alone was assumed. Certain concrete cases of pure probability can be found, for instance, in games of chance. Suppose we take $5x$ as meaning 5 red cards, and the various quantities of $y$ as meaning so many black cards. If increasing quantities of black cards were mixed with a fixed quantity of red cards, and if the whole pack were then dealt out in pairs, the probable number of mixed pairs and the probability of each black card meeting a red card would work out in every case in the manner just described.

Let us take a still more concrete case. If in a certain community there were a thousand women all of marriageable age, and all desiring to be married, and if in this same community there were only one man of marriageable age also desiring to be married, and if furthermore polygamy were prohibited, the probability that this one man would, within a measurable

[1] Only odd numbers of $y$ may be taken, otherwise all will not be paired off.

period of time, be married, would amount, for practical purposes, to certainty. The same would be true of any small number of men. But if the number of men increased considerably and amounted, say to 500, we should not be at all certain that all of them would be married off within a given time. If the number of marriageable men reached 1000, the chances of marriage for each man, tho good, would be much less than before. If the number of men increased further to 2000, 3000, 4000, etc., each man's chances of marriage would gradually diminish, while the chances of each woman would increase.

Here some one will protest that human factors are being neglected. Men and women have qualities which are sometimes better and sometimes worse; in one instance there may be more likelihood of attachment than in another; in one community social life gives to each sex more opportunities to meet the other than in another. All this is true and even more. Despite this, however, the element of chance still remains, and we are justified in our conclusion that as the number of either sex increases, the number of the other remaining constant, the probability that any one of the first will be married diminishes.

Further study will convince us that the law of probability underlies most, if not all, natural phenomena, and where this is true when two or more factors are under consideration, the law of diminishing returns must hold true. Ever since the time of Laplace, who, at the end of the eighteenth century, was the first to make a systematic study of the theory of probability and its applications, more and more attention has been given to its applicability to science. Its real significance, however, was not apparent until the work of Professor Willard Gibbs of Yale University. In his *Statistical Mechanics* he showed that the second law of thermodynamics was only a deduction from the laws of probability, that in fact all physical and chemical laws which are concerned with molecular action are likewise deductions from the same laws. There are millions of molecules in any mass, moving about at a tremendous rate, going in all directions, and making in-

numerable collisions; yet at any state of pressure and temperature, each molecule has an average mean velocity, there is on the whole an average direction and an average number of collisions.

The laws of chemical equilibrium, including the law of mass action, are corollaries of these laws of probability as applied to molecular activity. As this branch of chemistry is very technical, and as it would take us too far afield to go into detail, we cannot stop to derive these laws. Suffice it to say that when two substances react, the extent and the rate of the reaction is governed by the active mass of each substance. As the reaction proceeds, the rate gradually diminishes until an equilibrium is reached. Because of these laws, an increase of either one of the substances in reaction will increase the rate and extent of the reaction but not in proportion to the increase. For example, if we bring together ethyl alcohol and acetic acid, they interact to produce ethyl acetate and water. If we take these in equivalent ratio, action will cease when two-thirds of each is transformed into ethyl acetate and water. If the ethyl alcohol and acetic acid are brought together in the ratio of three to one, the action will go on until nine-tenths of the acetic acid is converted.

Enough has been said to show that diminishing returns is not an isolated phenomenon. Let us take an example from agriculture to which the law was first applied. The three important elements of plant food in the soil are potassium, nitrogen, and phosphorus. For proper plant growth these must exist in a certain ratio. If there is too little of any one element, the soil will not yield sufficient produce. If then we add some of this element to the soil in the form of a fertilizer, we shall have an increase of product. Up to a certain limit an increase in this element will bring an increase in the crop, but again not in proportion to the increase of the element. The following table, based upon the results of actual tests at the Rothamstead station, will illustrate this point:

| | | Average yield in bu. for 8 years | Gain for 43 lbs. of nitrogen |
|---|---|---|---|
| Plot 5 | mixed minerals alone | 19 | |
| 6 | " " and 43 lbs. nitrogen | $27\frac{7}{8}$ | $8\frac{7}{8}$ |
| 7 | " " " 86 " " | $35\frac{1}{2}$ | $7\frac{5}{8}$ |
| 8 | " " " 129 " " | $36\frac{7}{8}$ | $1\frac{3}{8}$ |
| 16 | " " " 172 " " | $37\frac{1}{2}$ | $\frac{5}{8}$ |

There is here a diminishing return from successive applications of nitrogen.

In many cases it is more difficult to trace the cause of diminishing returns. Yet in every case we find chance as one of the factors. We saw before that in a community where there were few men and many women, every man would be fairly certain to meet a woman and enter into the relation of marriage. We can apply this to the three factors, land, labor, and capital. If the number of the units of any one of these is small, each unit will be more likely to meet units of the other factors. If, to take a concrete instance, there were only one plow in the community, this plow would combine with a large number of labor and land units, and it would be sure to meet some of these units all the time. If the number of plows became inordinately large, each plow could not be used as much as before, and plows would be idle a great part of the time. The important fact in each of these cases is the probability of idleness. A plow is capital, and an increase of plows is an increase of capital. The second plow would add a great deal to the product, and so also would the third and fourth, but not quite so much. The same applies to an increase of labor or land.

We can examine this fact more closely if we take an example from the manufacturing industry. Suppose a laborer is employed at a milling machine, milling the end of a steel rod. His task can be analyzed into a series of motions, which consist of those made to operate levers, to adjust the rod, take it out, and put another in its place. The laborer is capable of making a certain number of motions within a given time. If he operates only one milling machine, his motions can be adjusted almost perfectly to the milling process; that is to say, within a given time, the motions required to be made

will combine with the machine process to give a maximum result for each machine. If now, he operates two milling machines, the combinations of muscular motions with the processes of the two machines will be more effective from the point of view of total product, but less effective for each machine. In other words, there is a diminishing return for each increase in the number of milling machines. This will be true, whether the constant be the number of men or the number of motions each man is capable of performing.

In view of the preceding, it is hardly possible to deny that the law of diminishing returns is universally valid. If criticism is at all possible, it can refer only to the applications which economists have made. Yet it is evident that valid applications can and must be made. As we are not here concerned with the subject matter of economics, but rather with its fundamental notions, no attempt will be made to apply the law.

To those who might object that what has been said is not physical fact, and can have no foundation in positive science, I should point out again that all the laws of science may be said to rest on probability. The law of probability is as much a part of the eternal nature of things as any of the laws of physics or chemistry. A protest might properly be made if there were here any attempt to set up a permanent scale of probabilities with reference to any of the factors of production. The differences between the returns from one quantity of any factor and an increased quantity of the same factor may be reduced to a minimum, but physical fact requires that some difference remain.

JULIUS DAVIDSON.